The Two Pillars Unified

From Buhlmann and Redington to Decision Governance

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The Problem

1.6×

excess decision variance

in identical risk strata

The Observation

- Same RAF score
- Same diagnoses
- Same predicted costs
- **Different actions**

$$\sigma^2 decision \gg \sigma^2 state$$

Decision variance far exceeds what estimation variance alone can explain

What CMS Does

CMS prices patient condition (state)

Everything not explained by state is treated as "noise"

But is all of that variance really random?

The Hidden Assumption

CMS Assumed Model:

$$Var(Y \mid RAF) = \sigma^2 state + \sigma^2 noise$$

Implicit: $\sigma^2 decision = 0$

Observed Reality:

$$Var(Y \mid RAF) = \sigma^2 state + \sigma^2 decision + \sigma^2 noise$$

The 1.6× lives here ↑

Classical credibility (Bühlmann 1967) assumes:

"Conditional on state, actions are stable or homogeneous."

But they're not. Provider decisions create variance beyond patient state.

The Bühlmann Completion

Extending classical credibility to the decision layer

$$\hat{Y}_{g,p} = \bar{Y} + Z_s \cdot (\bar{Y}_g - \bar{Y}) + Z_d \cdot (\bar{Y}_{g,p} - \bar{Y}_g)$$

Overall + State Credibility + Decision Credibility

$\bar{Y}$	$Z_s \cdot (\bar{Y}_g - \bar{Y})$	$Z_d \cdot (\bar{Y}_{g,p} - \bar{Y}_g)$
Baseline Overall mean	State Credibility Classical Bühlmann	Decision Credibility NEW — The completion

When $Z_d \to 0$: recovers classical Bühlmann exactly. This is a strict extension.

"Bühlmann stabilized estimation. We complete the theory by stabilizing decisions."

Case Study: Provider Variation

State 3: HCC 1.2-1.8 • Same risk scores, different providers

Provider	n	PMPM	Deviation
A (Conservative)	650	$980	**−$170**
B (Moderate)	800	$1,120	−$30
C (Aggressive)	550	$1,420	**+$270**

45%

PMPM Spread

$980 → $1,420

Same patients. Same HCC scores. 45% cost difference. That's σ^2decision.

Dual Credibility for Provider C: Zd = 0.993 → Deviation is signal, not noise

Classical Bühlmann would miss the $270 decision-induced deviation entirely

σ^2decision identified via within-state provider dispersion

Pillar 1: Bühlmann Credibility (1967)

What It Controls

$\hat{\mu} = Z \cdot \bar{X} + (1-Z) \cdot \mu_0$

- Stabilizes: Estimates
- Controls: σ^2state
- Domain: Learning
- Shrinks noisy signals toward prior

What It Does NOT Control

$\sigma^2(\text{decision} \mid \text{state})$

- Decision sensitivity $|\partial a/\partial x|$
- Action magnitude
- Execution admissibility

"How much should I trust this signal?"

Pillar 2: Redington Immunization (1952)

What It Controls

$DA = DL, \quad CA \geq CL$

- Stabilizes: Cashflows
- Controls: Sensitivity
- Domain: Execution
- Small shocks don't matter

What It Does NOT Control

$\sigma^2(\text{decision} \mid \text{state})$

- State learning
- Decisions as dynamic objects
- Assumes decisions are manual

"How do I survive the shock?"

The Unified Geometry - HVS

$$V(R, C; k) = R^k / C^{(1-k)}$$

This surface serves dual roles:

As STATE

A belief surface

Where you are

(Bühlmann domain)

As DECISION

A Lyapunov function

How you move

(Redington domain)

Same geometry. Different roles. Unified governance.

The Hidden Third Pillar

Black-Scholes (1973) was decision governance all along

What Black-Scholes Does

- State: underlying price S
- Decision: hedge ratio Δ
- Geometry: option value surface
- Constraint: no-arbitrage
- **Result: P&L variance controlled**

Our HVS Mapping

Black-Scholes	HVS
Option surface	$V = R^k/C^{(1-k)}$
No-arbitrage	$V \geq 1$
Gamma (γ)	k
Delta (Δ)	Zd

BS locks hedging decisions so state noise doesn't explode into P&L variance.

Finance never generalized this. HVS does.

The Complete Governance Stack

Layer	Tool	Symbol	Controls	Source
Estimation	Credibility	Zs	Var(state)	Bühlmann
Decision slope	Duration	Zd	$\lvert\partial a/\partial x\rvert$	Redington
Decision curve	Convexity	k	$\lvert\partial^2 a/\partial x^2\rvert$	Redington
Execution	Membrane	$V \geq 1$	Admissibility	Redington

Variance Collapse (Corollary)

$$\sigma^2 \text{decision} \leq Zd^2 \cdot L^2 \nabla V \cdot \sigma^2 \text{state}$$

The 1.6× excess variance represents unbounded $\lvert\partial a/\partial x\rvert$. Governance collapses it structurally.

Bifurcation vs Drift

Governance doesn't freeze the system — it stabilizes while remaining adaptive

BIFURCATION

Same state → different actions

- Fake spread from noise
- Chaos, not learning
- Burns patients/money during learning
- Cannot contract on outcomes

DRIFT

True optimal moves over time

- Real movement from evidence
- Adaptation, not chaos
- Learning tracks the moving center
- System stays responsive

LOOP A: Learning

"Where is the truth now?"

Allows drift • Tracks moving center

LOOP B: Governance

"How much can decisions move?"

Kills bifurcation • Collapses branches

Why This Matters

For 70 years, the pillars stood apart.

When decisions were **manual and slow**, the gap didn't matter.
Humans absorbed instability through judgment.

When decisions became **algorithmic and fast**,
instability re-entered through the decision operator—
precisely where no governance existed.

HVS closes that loop.

Actuarial science was always the science of decision stability.